A Note on Interference in Random Networks

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Abstract

The (maximum receiver-centric) interference of a geometric graph (von Rickenbach *et al.* (2005)) is studied. It is shown that, with high probability, the following results hold for a set, V, of n points independently and uniformly distributed in the unit d-cube, for constant dimension d: (1) there exists a connected graph with vertex set V that has interference $O((\log n)^{1/3})$; (2) no connected graph with vertex set V has interference $o((\log n)^{1/4})$; and (3) the minimum spanning tree of V has interference $\Theta((\log n)^{1/2})$.

1 Introduction

Von Rickenbach *et al.* [8, 9] introduce the notion of (maximum receiver-centric) interference in wireless networks and argue that topology-control algorithms for wireless networks should explicitly take this parameter into account. Indeed, they show that the minimum spanning tree, which seems a natural choice to reduce interference, can be very bad; there exists a set of node locations in which the minimum spanning tree of the nodes produces a network with maximum interference that is linear in the number, n, of nodes, but a more carefully chosen network has constant maximum interference, independent of n. These results are, however, *worst-case*; the set of node locations that achieve this are very carefully chosen. In particular, the ratio of the distance between the furthest and closest pair of nodes is exponential in the number of nodes.

The current paper continues the study of maximum interference, but in a model that is closer to a typical case. In particular, we consider what happens when the nodes are distributed uniformly, and independently, in the unit square. This distribution assumption can be used to approximately model the unorganized nature of ad-hoc networks and is commonly used in simulations of such networks [10]. Additionally, some types of sensor networks, especially with military applications, are specifically designed to be deployed by randomly placing (scattering) them in the deployment area. This distribution assumption models these applications very well.

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Our results show that the maximum interference, in this case, is very far from the worst-case. In particular, for points independently and uniformly distributed in the unit square, the maximum interference of the minimum spanning tree grows only like the square root of the logarithm of the number of nodes. That is, the maximum interference is *not even logarithmic* in the number of nodes. Furthermore, a more carefully chosen network topology can reduce the maximum interference further still, to the cubed root of the logarithm of n.

1.1 The Model

Let $V = \{x_1, \dots, x_n\}$ be a set of n points in $\mathbb{R}^d$ and let $G = (V, E)$ be a simple undirected graph with vertex set V. The graph G defines a set, $B(G)$, of closed balls $B_1, \dots, B_n$, where B_i has center x_i and radius

$$r_i = \max\{\|x_i x_j\| : x_i x_j \in E\} \ .$$

(Here, and throughout, $\|xy\|$ denotes the Euclidean distance between points x and y.) In words, B_i is just large enough to enclose all of x_i's neighbours in G. The *(maximum receiver-centric) interference* at a point, x, is the number of these balls that contain x, i.e.,

$$I(x, G) = |\{B \in B(G) : x \in B\}| \ .$$

The *(maximum receiver-centric) interference* of G is the maximum interference at any vertex of G, i.e.,

$$I(G) = \max\{I(x, G) : x \in V\} \ .$$

Figure 1 shows an example of a geometric graph G and the balls $B(G)$. Each node, x, is labelled with $I(x, G)$.

One of the goals of network design is to build, given V, a connected graph $G = (V, E)$ such that $I(G)$ is minimized. Thus, it is natural to consider interference as a property of the given point set, V, defined as

$$I(V) = \min\{I(G) : G = (V, E) \text{ is connected}\} \ .$$

A *minimum spanning tree* of V is a connected graph, $MST(V)$, of minimum total edge length. Minimum spanning trees are a natural choice for low-interference graphs. The purpose of the current paper is to prove the following results (here, and throughout, the phrase *with high probability* means with probability that approaches 1 as $n \to \infty$):

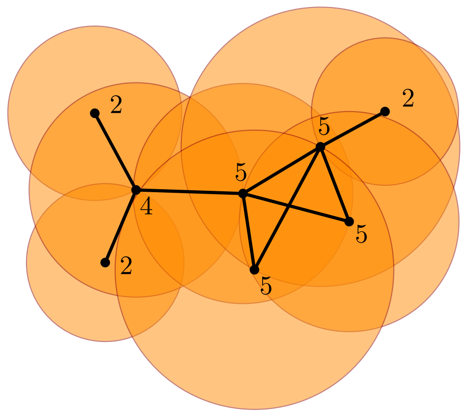

Figure 1: A geometric graph G with $I(G) = 5$.

Theorem 1. *Let V be a set of n points independently and uniformly distributed in $[0,1]^d$. With high probability,*

1. $I(MST(V)) \in O((\log n)^{1/2})$;
2. $I(V) \in O((\log n)^{1/3})$, *for* $d \in \{1,2\}$; *and*
3. $I(V) \in O((\log n)^{1/3}(\log\log n)^{1/2})$, *for* $d \geq 3$.

Theorem 2. *Let V be a set of n points independently and uniformly distributed in $[0,1]^d$. With high probability,*

1. $I(MST(V)) \in \Omega((\log n)^{1/2})$
2. $I(V) \in \Omega((\log n)^{1/4})$.

1.2 Related Work

This section surveys previous work on the problem of bounding the interference of worst-case and random point sets. A summary of the results described in this section is given in Figure 2. In the statements of all results in this section, $|V| = n$.

The definition of interference used in this paper was introduced by von Rickenbach *et al.* [8] who proved upper and lower bounds on the interference of one dimensional point sets:

Theorem 4 (von Rickenbach *et al.* 2005)**.** *For any $d \geq 1$, there exists $V \subset \mathbb{R}^d$ such that $I(V) \in \Omega(n^{1/2})$.*

The point set, V, in this lower-bound consists of any sequence of points $x_1, \ldots, x_n$, all on a line, such that $\|x_{i+1}x_i\| \leq (1/2)\|x_i x_{i-1}\|$, for all $i \in \{2, \ldots, n-1\}$. That is, the gaps between consecutive points decrease exponentially.

This lower bound is matched by an upper-bound:

Theorem 5 (von Rickenbach *et al.* 2005)**.** *For all $V \subset \mathbb{R}$, $I(V) \in O(n^{1/2})$.*

The upper bound in Theorem 5 is obtained by selecting $n^{1/2}$ vertices to act as *hubs*, connecting the hubs into any connected network and then having each of the remaining nodes connect to its nearest hub. This idea was extended to two and higher dimensions by Halldórsson and Tokuyama [3], by using a special type of $(n^{-1/2})$-net as the set of hubs:

Theorem 6 (Halldórsson and Tokuyama 2008)**.** *For all $V \subset \mathbb{R}^d$,*

1. $I(V) \in O(n^{1/2})$ *for* $d = 2$; *and*
2. $I(V) \in O((n \log n)^{1/2})$, *for* $d \geq 3$.

Several authors have shown that the interference of a point set is related to the (logarithm of) the ratio between the longest and shortest distance defined by the point set. In particular, different versions of the following theorem have been proven by Halldórsson and Tokuyama [3]; Khabbazian, Durocher, and Haghnegahdar [4]; and Maheshwari, Smid, and Zeh [6]:

Theorem 7 (Halldórsson and Tokuyama 2008; Khabbazian, Durocher, and Haghnegahdar 2011; Maheshwari, Smid, and Zeh 2011)**.** *For any constant $d \geq 1$ and for all $V \subset \mathbb{R}^d$, $I(V) = O(\log D)$, where $D = \max\{\|xy\| : \{x,y\} \subseteq V\}/\min\{\|xy\| : \{x,y\} \subseteq V\}$.*

At least two of the proofs of Theorem 7 proceed by showing that $I(MST(V)) = O(\log D)$. A strengthening of this theorem is that the numerator in the definition of D can be replaced with the length of the longest edge in $MST(V)$ [4, 6].

Theorem 7 suggests that point sets with very high interference are unlikely to occur in practice. This intuition is born out by the results of Kranakis *et al.* [5], who show that high interference is unlikely to occur in random point sets in one dimension:

Theorem 8 (Kranakis *et al.* 2010)**.** *Let V be a set of n points independently and uniformly distributed in $[0,1]$. Then, with high probability, $I(MST(V)) \in \Theta((\log n)^{1/2})$.*

Note that, in this one-dimensional case, the minimum spanning tree, $MST(V)$, is simply a path that connects the points of V in order, from left to right. Taken together, Part 1 of Theorems 1 and 2 generalize Theorem 8 to arbitrary constant dimensions $d \geq 1$.

In higher dimensions, Khabbazian, Durocher, and Haghnegahdar [4] use their version of Theorem 7 to show that minimum spanning trees of random point sets have at most logarithmic interference.

Theorem 9 (Khabbazian, Durocher, and Haghnegahdar 2011)**.** *Let V be a set of n points independently and uniformly distributed in $[0,1]^d$. Then, with high probability, $I(MST(V)) \in O(\log n)$.*

Ref.	Dimension	Statement
[8]	$d \geq 1$	there exists V s.t. $I(V) \in \Omega(n^{1/2})$
[8]	$d = 1$	for all V, $I(V) \in O(n^{1/2})$
[3]	$d = 2$	for all V, $I(V) \in O(n^{1/2})$
[3]	$d \geq 3$	for all V, $I(V) \in O((n \log n)^{1/2})$
[5]	$d = 1$	for V i.u.d. in $[0,1]$, $I(MST(V)) \in \Theta((\log n)^{1/2})$ w.h.p.
[4]	$d \geq 2$	for V i.u.d. in $[0,1]^d$, $I(MST(V)) \in O(\log n)$ w.h.p.
Here	$d \geq 1$	for V i.u.d. in $[0,1]^d$, $I(MST(V)) \in \Theta((\log n)^{1/2})$ w.h.p.
[5, 8]	$d = 1$	for V i.u.d. in $[0,1]$, $I(V) \in \Omega((\log n)^{1/4})$ w.h.p.
Here	$d \geq 1$	for V i.u.d. in $[0,1]^d$, $I(V) \in \Omega((\log n)^{1/4})$ w.h.p.
Here	$d \in \{1,2\}$	for V i.u.d. in $[0,1]^d$, $I(V) \in O((\log n)^{1/3})$ w.h.p.
Here	$d \geq 3$	for V i.u.d. in $[0,1]^d$, $I(V) \in O((\log n)^{1/3}(\log\log n)^{1/2})$ w.h.p.

Figure 2: Previous and new results on interference in geometric networks.

Part 1 of Theorem 1 improves the upper bound in Theorem 9 to $O((\log n)^{1/2})$ and Part 1 of Theorem 2 gives a matching lower bound.

The second parts of Theorems 1 and 2 show that minimum spanning trees do not minimize interference, even for random point sets. For random point sets, one can construct networks with interference $O((\log n)^{1/3})$ and the best networks have interference in $\Omega((\log n)^{1/4})$.

The remainder of this paper is devoted to proving Theorems 1 and 2. For ease of exposition, we only present these proofs for the case $d = 2$ though they generalize, in a straightforward way, to arbitrary (constant) dimensions. Due to space constraints, some proofs are omitted from this version of the paper. All proofs can be found in the preprint version [2].

2 Proof of the Upper Bounds (Theorem 1)

In this section, we prove Theorem 1. However, before we do this, we state a slightly modified version of Theorem 7 that is needed in our proof.

Lemma 1. *Let $V \subset \mathbb{R}^d$, let $r > 0$, and let $MST^r(V)$ denote the subgraph of $MST(V)$ containing only the edges whose length is in $(r, 2r]$. Then $I(MST^r(V)) \in O(1)$.*

Proof. (This proof is similar to the proof of Lemma 3 in Ref. [6].) Let x be any point in $\mathbb{R}^d$ and let B the set of all balls in $B(MST^r(V))$ that contain x so that, by definition $I(x, MST^r(V)) = |B|$. All the centers of balls in B are contained in a ball of radius $2r$ centered at x. Therefore, a simple packing argument implies that there exists a ball, b, of radius $r/2$ that contains at least $|B|/5^d$ centers of balls in B. (5^d is the volume of a ball of radius $5r/2$ divided by the volume of a ball of radius $r/2$.) The center of each of these ball is the endpoint of an edge of length at most $2r$. The other endpoints of these edges are all contained in a ball of radius $5r/2$ centered around b. The same packing argument shows that we can find a ball of radius $r/2$ that contains at least $|B|/(5 \cdot 6)^d$ of these other endpoints.

We claim that this implies that $|B|/30^d < 2$ (so $|B| < 2 \cdot 30^d$). Otherwise, $MST(V)$ contains two edges, $x_i x_j$ and $x_k x_\ell$, each of length greater than r and such that $\|x_i x_k\| \leq r$ and $\|x_j x_\ell\| \leq r$. But this contradicts the minimality of $MST(V)$, since one could replace $x_i x_j$ with one of $x_i x_k$ or $x_j x_\ell$ and obtain a spanning tree of smaller total edge length. We conclude that $|S_i| < 2 \cdot 30^d$, and this completes the proof. □

Note that Lemma 1 implies Theorem 7, since it implies that we can partition the edges of $MST(V)$ into $\lceil \log_2 D \rceil$ classes, based on length, and each class will contain only a constant number of edges.

We are ready to prove Parts 2 and 3 of Theorem 1. The sketch of the proof is as follows: We partition $[0,1]^d$ into equal cubes of volume $1/nt$, for some parameter t to be chosen later. Using Chernoff's bounds, we show that each cube contains $O((\log n)^{2/3})$ points so that the points within each cube can be connected, using the results of Halldórsson and Tokuyama, with maximum interference $O((\log n)^{1/3})$. Next, the cubes are connected to other cubes by selecting one point in each cube and connecting these selected points with a minimum spanning tree. Lemma 1 is then used to show that this minimum spanning tree has maximum interference $O((\log n)^{1/3})$. Without further ado, we present:

Proof of Theorem 1, Parts 2 and 3. Partition $[0,1]^2$ into square *cells* of area $1/nt$ for some value t to be specified later. Let N_i denote the number of points that are contained in the ith cell. Then N_i is binomial with mean $\mu = 1/t$. Recall Chernoff's Bounds [1] on the tails of binomial random variables:

$$\Pr\{N_i \geq (1+\delta)\mu\} \leq \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^\mu .$$

In our setting, we have,

$$\begin{aligned}\Pr\{N_i \geq k\} &= \Pr\{N_i \geq kt\mu\} \\ &\leq \left(\frac{e^{kt}}{(kt)^{kt}}\right)^{1/t} \\ &= \frac{e^k}{(kt)^k} \\ &\leq \frac{1}{t^k} \qquad \text{for } k \geq e \\ &\leq \frac{1}{n^{c+2}} \ ,\end{aligned}$$

for $t = 2^{(\log n)^{1/3}}$ and $k = (c+2)(\log n)^{2/3}$.

Note that the number of cells is no more than $nt \leq n^2$, for sufficiently large n. Therefore, by the union bound, the probability that there exists any cell containing more than k points is at most n^{-c}.

Within each non-empty cell, we apply Theorem 6 to connect the vertices in the ith cell into a connected graph G_i with $I(G_i) = O(\sqrt{N_i})$.[1] In fact, a somewhat stronger result holds, namely that $\max\{I(x, G_i) : x \in \mathbb{R}^2\} = O(\sqrt{N_i})$. Notice that each edge in G_i has length at most $\sqrt{2/nt}$. Stated another way, in $\bigcup_i G_i$, any point, x, receives interference only from cells within distance $\sqrt{2/nt}$ of the cell containing x. There are only 25 such cells, so

$$\max\left\{I\left(x, \bigcup_i G_i\right) : x \in \mathbb{R}^2\right\} = O(\sqrt{k}) = O((\log n)^{1/3})$$

with high probability.

Thus far, the points within each cell are connected to each other and the maximum interference, over all points in $\mathbb{R}^2$, is $O(\sqrt{k})$. To connect the cells to each other, we select one point from each non-empty cell and connect these using a minimum spanning tree, T. What remains is to show that the additional interference caused by the addition of the edges in T does not exceed $O((\log n)^{1/3})$.

Suppose that $I(x, T) = r$, for some point $x \in \mathbb{R}^2$. There are at most 9 vertices in T whose distance to x is less than $1/\sqrt{nt}$. Therefore, by Lemma 1, T must contain an edge of length at least $c2^r/\sqrt{nt}$, for some constant $c > 1$.

A well-known property of minimum spanning trees is that, for any edge $x_i x_j$ in T, the open ball with diameter $x_i x_j$ does not contain any vertices of T. In our setting, this means that there is an open ball, B, of radius $c2^r/2\sqrt{nt}$ such that every cell contained in B contains no point of V. Inside of B is another empty ball B' of radius $c2^r/(2\sqrt{nt}) - \sqrt{2/nt}$ whose center is also the center of some cell.

[1] This is where the discrepancy between Parts 2 and 3 of the theorem occurs. For $d \geq 3$, Theorem 6 only guarantees $I(G_i) = O(\sqrt{N_i \log N_i})$.

At least one quarter of the area of B' is contained in $[0,1]^2$, so the number of cells completely contained in B' is at least $\pi c^2 2^{2r}/16 - O(2^r/\sqrt{nt})$. By decreasing c slightly, and only considering r larger than a sufficiently large constant, r_0, we can simplify this number of cells to $\pi c^{2r}/16$.

For a fixed ball B', the probability that the $c\pi 2^{2r}/16$ cells defined by B' are empty of points in V is at most

$$\begin{aligned}p &\leq (1 - c\pi 2^{2r}/16nt)^n \\ &\leq \exp(-c\pi 2^{2r}/16t) \\ &\leq 1/n^{2+c'} \ ,\end{aligned}$$

for $r \geq \log(16/c\pi) + \log t + \log(2 + c') + \log\ln n$. By the union bound, the probability that there exists any such B' is at most $pnt \leq 1/n^{c'}$. Since we can choose $r \in O(\log t + \log\log n) = O((\log n)^{1/3})$, this completes the proof. □

The proof of Part 1 of Theorem 1 is just a matter of reusing the ideas from the previous proof of Parts 2 and 3.

Proof of Theorem 1, Part 1. Let x be any point in $\mathbb{R}^2$. We partition the balls in $B(MST(V))$ that contain x into three sets:

1. the set B_0 of balls having area at most $1/nt$;
2. the set B_1 of balls having area in the range $[1/nt, (c\log n)/n]$; and
3. the set B_2 of balls having area greater than $(c\log n)/n$.

In this proof, the parameter $t = 2^{(\log n)^{1/2}}$.

The set B_0 consists of points contained in a ball of area $1/nt$ centered at x. Exactly the same argument used in the first part of the previous proof shows that, with high probability, every such ball contains $O((\log n)^{1/2})$ points, so

$$|B_0| \in O((\log n)^{1/2}) \ .$$

The set B_1 consists of balls whose radii are in the range $[\sqrt{1/\pi nt}, \sqrt{(c\log n)/\pi n}]$. Lemma 1 shows that the number of these balls is

$$\begin{aligned}|B_1| &\in O\left(\log\left(\frac{\sqrt{(c\log n)/\pi n}}{\sqrt{1/\pi nt}}\right)\right) \\ &= O(\log\log n + \log t) \\ &= O((\log n)^{1/2}) \ .\end{aligned}$$

Finally, any edge in the set B_2 implies the existence of an empty ball, with center in $[0,1]^2$, having area $c\log n/n$. The second part of the previous proof shows that the probability that such a ball exists is $O(n^{-c})$. Therefore, with high probability,

$$|B_2| = 0 \ .$$

□

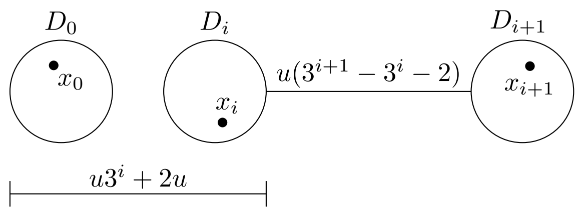

Figure 4: The ball centered at x_i that contains x_{i+1} also contains x_0.

3 Proof of The Lower Bounds (Theorem 2)

In this section, we prove the lower bounds in Theorem 2. We define a *Zeno configuration* as follows (see Figure 3): A Zeno configuration of size k, centered at a point, x, is defined by a set of $k+1$ balls. The construction starts with disjoint balls $D_0, \dots, D_{k-1}$, each having radius u. The ball D_0 is centered at x. The center of D_i, $i \in \{1, \dots, k-1\}$ is at $x + (u3^i, 0)$. A final large ball, D, of radius $r = u3^k$ is centered at x and contains all other balls. A Zeno configuration occurs at location x in a point set V when D contains exactly k points of V and these occur with exactly one point in each ball D_i.

The following lemma shows that a Zeno configuration in V causes high interference in $MST(V)$.

Lemma 2. *If V contains a Zeno configuration of size k, $I(MST(V)) \geq k-1$.*

Proof. Let x_i, $i \in \{0, \dots, k-1\}$, denote the point of V contained in D_i. Note that, for $i \in \{1, \dots, k-1\}$ the closest point to x_i in V is x_{i-1}. Since $MST(V)$ contains the nearest-neighbour graph, this implies that $MST(V)$ contains the edges $x_i x_{i+1}$ for all $i \in \{0, \dots, k-2\}$. See Figure 4 for what follows. We claim that, for all $i \in \{0, \dots, k-2\}$, the ball B_i centered at x_i that contains x_{i+1} also contains x_0. This is clearly true for $i = 0$ and $i = 1$. Next, note that

$$\|x_i x_0\| \leq u(3^i + 2) \ .$$

On the other hand, for $i \geq 2$,

$$\|x_i x_{i+1}\| \geq u(3^{i+1} - 3^i - 2) = 2u3^i - 2u \geq u(3^i + 7) > \|x_i x_0\| \ .$$

Therefore, $I(x_0, MST(V)) \geq k-1$. □

The next lemma shows that a Zeno configuration causes high interference on any connected graph on vertex set V.

Lemma 3. *If V contains a Zeno configuration of size k, then $I(V) \geq \sqrt{k-1}$.*

Proof. Let G be any connected graph on V. Using the same notation as in the proof of Lemma 2, call a vertex, x_i, a *big one* if x_i is adjacent to any vertex x_j, with $j > i$, or x_i is adjacent to any vertex x not in D. The proof of Lemma 2 shows that every big one contributes to the interference at x_0. Therefore, if the Zeno configuration contains $\sqrt{k-1}$ or more big ones, then $I(x_0, G) \geq \sqrt{k-1}$ and there is nothing left to prove. Otherwise, note that each of $x_0, \dots, x_{k-2}$ is either a big one or adjacent to a big one. Therefore, there must be a big one, x_i, with degree at least $\sqrt{k-1} - 1$, so $I(x_i, G) \geq \sqrt{k-1}$. □

To prove Theorem 2, all that remains is to show a Zeno configuration of size $\Omega((\log n)^{1/2})$ occurs in V with high probability. We omit this proof due to space constraints.

4 Discussion

Summary. This paper gives new bounds on the maximum interference for graphs defined by points randomly distributed $[0,1]^d$. Minimum spanning trees have interference $\Theta((\log n)^{1/2})$, but better graphs exist; a strategy based on bucketing yields a graph with interference $O((\log n)^{1/3})$. No graph on such a point set has interference $o((\log n)^{1/4})$.

Open Problem. An obvious open problem is that of closing the gap between the upper bound of $O((\log n)^{1/3})$ and the lower bound of $\Omega((\log n)^{1/4}$. One strategy to achieve this would be to prove the following conjecture, which has nothing to do with probability theory:

Conjecture 1. *For any $V \subset \mathbb{R}^d$, $I(V) = O(\sqrt{I(MST(V))})$.*

A weaker version of this conjecture is due to Halldórsson and Tokuyama [3], who conjecture that $I(V) = O(\sqrt{\log D})$ where D is the ratio of the lengths of the longest and the shortest edges of $MST(V)$.

Unit Disk Graphs. Several of the references consider interference in the *unit disk graph model*, in which the graph G is constrained to use edges of maximum length $r(n)$. It is straightforward to verify that all of the proofs in this paper continue to hold in this model, when $r(n) \in \Omega(\sqrt{(\log n)/n})$. This is not an unreasonable condition; for i.u.d. points in $[0,1]^d$, it is known that $r(n) \in \Omega(\sqrt{(\log n)/n})$ is a necessary condition to be able to form a connected graph G [7].

Locally Computable Graphs. Khabbazian, Durocher, and Haghnegahdar [4] give a local algorithm, called LocalRadiusReduction, that is run at the nodes of a communication graph, $G = (V, E)$, and that reduces the number of edges of G. The resulting graph G' comes from a class of graphs that they denote as $\mathcal{T}(V)$. The

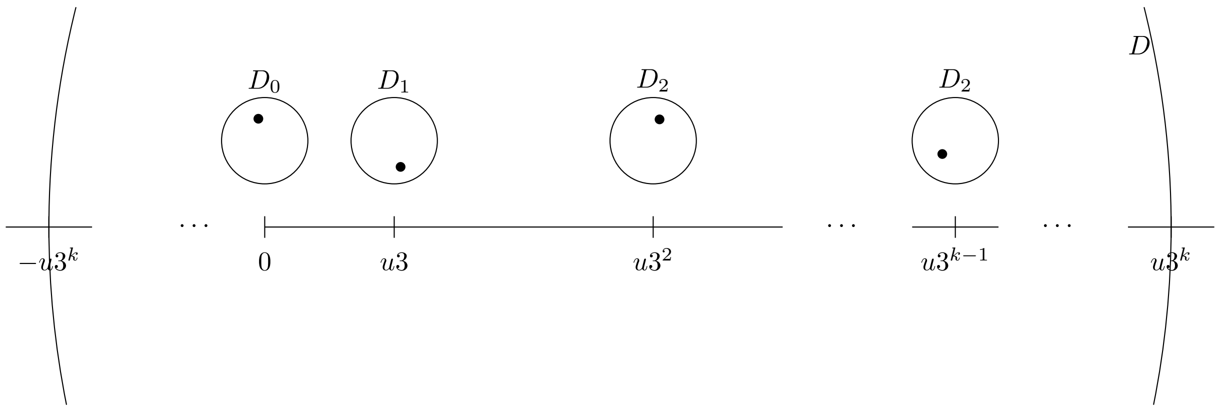

Figure 3: A Zeno configuration of size k.

class $\mathcal{T}(V)$ includes the minimum spanning tree of V and the graphs in this class share many of the same properties as the minimum spanning tree. In particular, the following result can be obtained by using the proof of Theorem 1 Part 1 and properties of the family $\mathcal{T}(V)$ [4, Theorem 3].

Theorem 3. *Let V be a set of n independently and uniformly distributed points in $[0,1]^d$ and let G be any graph in $\mathcal{T}(V)$. With high probability, $I(G) = O((\log n)^{1/2} + \log(\ell\sqrt{n}))$, where ℓ is the length of the longest edge in G.*

In particular, Theorem 3 implies that running the LocalRadiusReduction algorithm at the nodes of a unit disk graph with unit $r(n) \in O(2^{\sqrt{\log n}}/\sqrt{n})$ yields a connected graph with maximum interference $O((\log n)^{1/2})$.

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