COMP 251

MIDTERM I

2020

COMP 251 - 2020

YOUR NAME	YOUR ID#

Q1.

(i) $T_n \le T_{\lfloor n/2 \rfloor} + 1$

(ii) $T_n \le T_{\lfloor n/2 \rfloor} + T_{\lceil n/2 \rceil} + n - 1$

(iii) $T_n \le 7T_{n/2} + n^2$

Q2.

Compute $\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \begin{vmatrix} x_3 & x_4 \\ y_3 & y_4 \end{vmatrix}$ using Strassen's 7 multiplications. For the remaining two dot products, $x_1x_2 + y_1y_2$, $x_3y_3 + x_4y_4$, compute 4 products. The total is $7 + 4 = 11$.

Q3.

(i) $\lceil \log_2 42 \rceil = 6$

(ii) $\lceil \log_2 6! \rceil = 10$

(iii) $\lceil \log_2 \binom{8}{3} \rceil = \lceil \log_2 56 \rceil = 6$

(iv) $\lceil 9/\log_2 10 \rceil = 3$

Q4.

set $k \leftarrow \lfloor i/b_j \rfloor$.

$$N[i,j] \leftarrow \sum_{\ell=0}^{k} N[i-\ell k, j-1].$$

Q5.

(i) F

(ii) T

(iii) F

Q6.

(i) By two horizontal cuts, split the points into three sets of size $2n$ each. Color these points white, black and red. By the ham sandwich theorem, we can halve each of these sets by one hyperplane cut, leaving six sets of size n each.

(ii) 7

(iii) 45

(iv) 32

Q7.

$T_n \le T_{n/3} + T_{2n/3} + Cn$ for some constant C. The solution is $T_n \le O(n \log n)$ by observing that in a recursion tree, each of the $\Theta(\log n)$ levels requires work proportional to n. [One could also use induction.]

Q8.

A B C D E F G H I J K