

COMP 251
MIDTERM II
PROTOTYPE

SCORE

YOUR NAME

YOUR ID.

LUC DEVROYE

QUESTION 1

(i)

ABDEFC

(ii)

DFEBCA

(iii)

DBFEAC

(iv)

B

(v)

110110 0010

QUESTION 2

SELECT (R, t):

$x \leftarrow t, l \leftarrow R$

repeat forever: if $\text{left}[x] = \text{nil}$ then $r \leftarrow 1$
else $r \leftarrow 1 + \text{size}[\text{left}[x]]$

case $l < r$: $x \leftarrow \text{left}[x]$

$l = r$: return $\text{key}[x]$ and exit

$l > r$: $l \leftarrow l - r$

$x \leftarrow \text{right}[x]$

QUESTION 3

SMALLEST (R, t): We use an auxiliary binary heap H

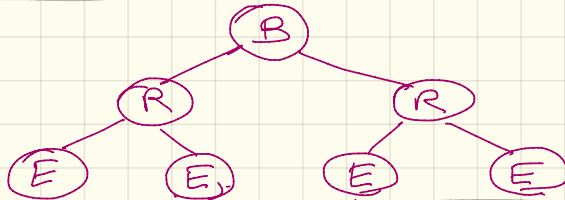
MAKEALL (H)

INSERT (t, H)

repeat R times: $x \leftarrow \text{DELETEMIN}(H)$
if $\text{left}[x] \neq \text{nil}$ then INSERT ($\text{left}[x], H$)
if $\text{right}[x] \neq \text{nil}$ then INSERT ($\text{right}[x], H$)
output $\text{key}[x]$

Note: At any point in time, $|H| \leq R+1$.

QUESTION 4



QUESTION 5 CHECK(t, a, y): Let MIN(t), MAX(t) find the minimum and maximum key in the subtree of t .
 If $y = \text{MAX}(\text{left}[t])$ or $\text{MIN}(\text{right}[t])$ or $a = \text{MAX}(\text{left}[y])$ or $\text{MIN}(\text{right}[y])$
 then return "true"
 else return "false"

QUESTION 6
 XXX: $b < \text{max}[\text{left}[t]]$
 YYY: "yes"
 ZZZ: contained($\text{right}[t], [a, b]$)

QUESTION 7

(i): T

(ii): T

(iii): T

(iv): F

(v): F

(vi): 6, 7, 8

(vii): smallest C
 middle B
 largest A

(viii): $1 + \frac{1}{2} + \dots + \frac{1}{n}$

(ix): $\frac{n+1}{2}$

(x): $\frac{2^{n+1}}{3}$

(xi): $2 \times 4^n - 1$